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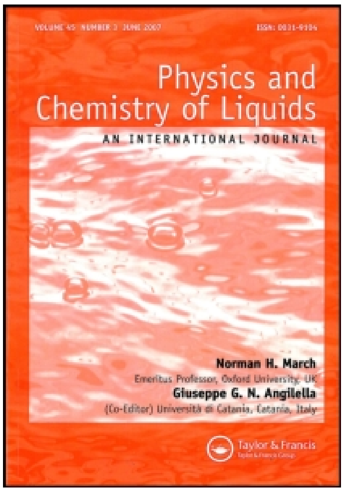

Physics and Chemistry of Liquids

Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713646857

Melting Curve of Wigner Electron Crystal

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To cite this Article Ferraz, A. , March, N. H. and Suzuki, M.(1978) 'Melting Curve of Wigner Electron Crystal', Physics and Chemistry of Liquids, 8: 3, 153 — 156

To link to this Article: DOI: 10.1080/00319107808084749

URL: http://dx.doi.org/10.1080/00319107808084749

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Phys. Chem. Liq., 1978, Vol. 8, pp. 153–156

Printed in Holland

Melting Curve of Wigner Electron Crystal

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(*Received March 8, 1978*)

The shape of the melting line of the Wigner crystal is discussed in the temperature-density plane. It is argued that near the critical density ρ^* for the Wigner transition at $T = 0$ the melting curve has the form

$$T \propto (\rho^* - \rho)^{1/2}.$$

For extreme low densities, on the other hand, the melting curve has as its asymptote the classical plasma melting line

$$T = 1.7 \times 10^{-5}\,\rho^{1/3}: \quad (\rho \text{ in units of } 10^{18}\ \text{cm}^{-3})$$

It seems that there should be a single maximum in the melting curve. The classical plasma line should provide an upper bound for the melting temperature, but is a poor bound away from the low density limit.

1 INTRODUCTION

The crystallization of degenerate electrons in a uniform positive background was argued to occur because of the mutual Coulomb repulsion, beyond a certain coupling strength, by Wigner.[1] Most attention has been focussed on the critical density ρ^* for the transition at $T = 0$. Even here, however, a wide spread of estimated values exists.[2]

That such crystallization occurs in the classical electron plasma was first demonstrated in a machine experiment by Brush, Sahlin and Teller.[3] They showed that when the ratio of the potential energy per particle e^2/r_s to the thermal energy $k_b T$ exceeded a certain value, crystallization occurred, as discussed also by Pollock and Hansen.[4]

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† Supported by CNPq and Depto de Fisica da UFPE (Brazil).

In this note, we shall give a semi-quantitative discussion of the melting line of the Wigner crystal and in particular we shall exhibit a relation between the melting curve and the classical plasma crystallization referred to above.

2 LOW DENSITY LIMIT

We can argue that the melting curve must take the form

$$T = \frac{e^2}{r_s} F\left(\frac{r_s}{a_0}\right) \tag{1}$$

a_0 being the Bohr radius $\hbar^2/me^2$, while r_s is the usual interelectronic spacing, related to the density ρ by

$$\rho = \frac{3}{4\pi r_s^3} \tag{2}$$

Eqs. (1) and (2) show that $\hbar^2\rho^{1/3}$ appears in the function F and therefore that as $\rho \rightarrow 0$, Planck's constant also disappears from the problem.

Thus, the classical result for the critical ratio Γ for crystallization

$$\Gamma = 155 = \frac{e^2}{r_s k_b T} \tag{3}$$

referred to above becomes the asymptote to the melting line (1) in the limit $\rho \rightarrow 0$, that is from (3) the asymptote to the Wigner melting curve is

$$T = 1.7 \times 10^{-5}\rho^{1/3} \tag{4}$$

and this is shown in Figure 1. We can say that the low density-low temperature form (4) of the melting curve is characteristic of electrons which are classical.

3 BEHAVIOUR NEAR CRITICAL DENSITY ρ^*

We turn now to the behaviour near the Wigner transition density ρ^* at $T = 0$. Along the transition line the equilibrium condition between the two phases is given by $F_m(\rho, T) = F_w(\rho, T)$ where F is the Helmholtz free energy. Thus

$$E_m(\rho, S) - TS_m = E_w(\rho, S) - TS_w, \tag{5}$$

where the subscripts m and w refer to the metal and Wigner phases respectively.

As $T \rightarrow 0$

$$S_m = Tf(\rho), \tag{6}$$

and

$$S_w \simeq \text{const } T^3 \tag{7}$$

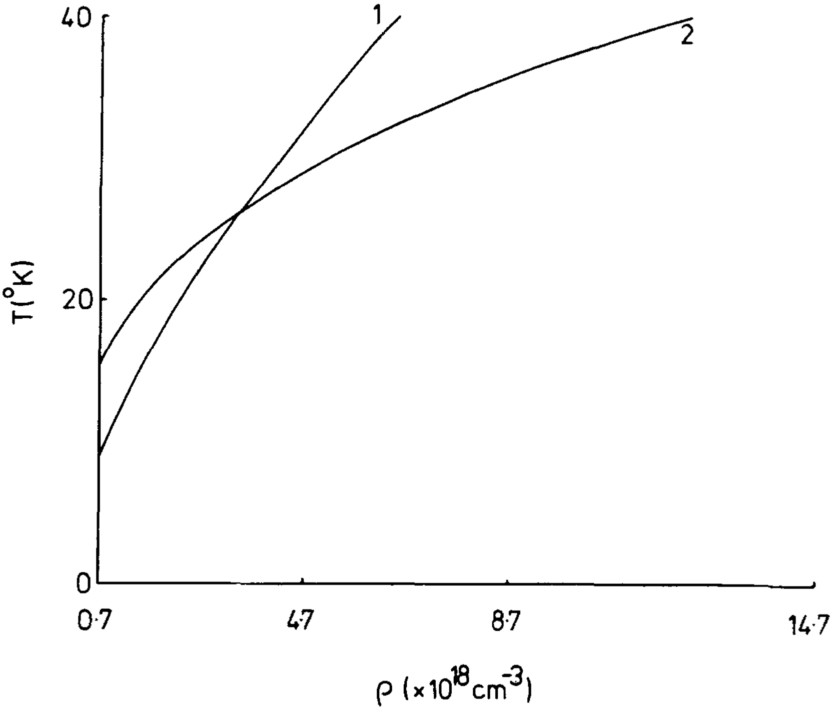

FIGURE 1 (1) Low density form of Classical plasma melting curve $T = 1.7 \times 10^{-5}\,\rho^{1/3}$. (2) "Degeneracy curve" $T = 1.1 \times 10^{-11}\,\rho^{2/3}$, along which the thermal de Broglie wavelength equals the interparticle spacing.

due to electron and phonon excitations respectively. Therefore in the neighbourhood of the Wigner transition point at $T = 0$, $\rho = \rho^*$

$$E_m(\rho^*, S_0) + (\rho - \rho^*)\frac{\partial E_m}{\partial \rho}(\rho^*, S_0) - E_w(\rho^*, S_0)$$

$$-(\rho - \rho^*)\frac{\partial E_w}{\partial \rho}(\rho^*, S_0) \simeq T^2 f(\rho^*) \tag{8}$$

Hence,

$$-(\rho - \rho^*)\left[\frac{\partial E_w}{\partial \rho}(\rho^*, S_0) - \frac{\partial E_m}{\partial \rho}(\rho^*, S_0)\right] \simeq T^2 f(\rho^*) \tag{9}$$

Thus we find a melting curve shown schematically in Figure 2, with

$$T \propto (\rho^* - \rho)^{1/2}, \tag{10}$$

near $T = 0$, $\rho = \rho^*$.

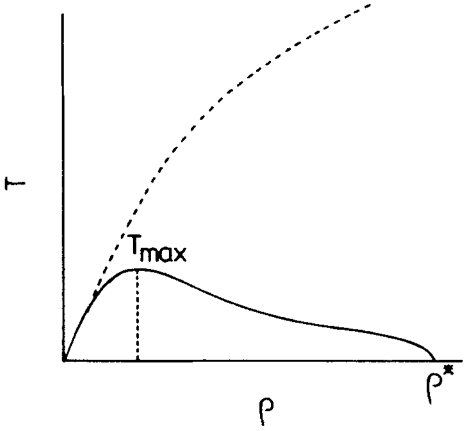

FIGURE 2 Schematic form of melting curve of Wigner crystal. Note

i) low density asymptote $\alpha\rho^{1/3}$ (broken curve)

ii) vertical slope at ρ^*.

$T^{\max}$ is expected to be but a small fraction of the classical (upper bound) curve.

4 SUMMARY

We believe that the classical melting line (4) is both an asymptote to the melting of the Wigner crystal at low densities and is also an upper bound. However, the bound will become very poor at temperatures such that the thermal de Broglie wavelength $\lambda \sim r_s$: this condition $\lambda = r_s$ being shown in Figure 1. The two curves shown cross around 25°K and marked departures from the asymptote will occur before that point as depicted in Figure 2. We expect that the maximum melting temperature will be quite low, perhaps in the helium range, but this is not established by our arguments. However, these make clear that the general shape of the melting curve is as in Figure 2.

Acknowledgments

One of us (N.H.M.) wishes to thank Professor D. J. Thouless for a helpful correspondence and another of us (A.F.) is grateful to Dr. P. J. Grout for his help.

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